The Movement of a Soluble Material During the Washing of a Bed of Packed Solids

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In the washing of a bed of packed solid particles, saturated with a fluid containing soluble materials, a number of complex transport phenomena may be involved. If the solid particles in the bed are porous, then the solute may be initially contained both within the solid particles and in solution contained in the interparticle voids. In order for the solute in the particle to be removed from the bed, it must move from the internal structure of the particle to the external surface of the particle and then into the fluid contained in the interparticle voids. This may involve molecular diffusion within the particle, adsorption and desorption on the internal surfaces of the particle, surface diffusion along the internal surfaces of the particle, and electrokinetic phenomena. In the idealized case of piston flow, the fluid in the interparticle voids will be removed by a simple displacement by the wash fluid. However, in any real bed the involved flow pattern, resulting from the complex geometry of the bed, will cause the entering fluid to mix with the fluid originally in the bed. In addition to this type of mixing, caused by the flow pattern around and among the particles in the bed, an additional mixing may occur owing to diffusion of the solute.

Although a complete mathematical description of the complex flow pattern in a bed of randomly oriented solid particles is not presently available, the overall movement of a solute, in a bed of nonporous granular material, has been described with a diffusionlike differential equation (*2, 4, 7, 9, 12, 15*), in which the molecular diffusion coefficient is replaced by a longitudinal dispersion coefficient which includes both the effect of molecular diffusion and the effect of the varying flow pattern in the bed.

While there have been numerous investigations of the washing of relatively simple beds of granular materials, few studies have been made with the more complex fibrous systems. There are several important differences between a bed of granular media and a bed of fibrous media. With spherical particles (granular media), the particles are always geometrically similar regardless of size, and with reasonable care the packing will also be similar. In the case of long cylindrical particles (fibrous media), the particles are not necessarily geometrically similar, since the length-to-diameter ratio may vary. It is also doubtful that geometrically similar packing is obtained when different size fibers are used. Another difference between a fiber system and a granular system is the porosity. Most granular media are incompressible and have a relatively low porosity of 0.35 to 0.7 which is fixed for any given bed. Fibrous systems, however, are compressible, and the porosity of a given bed can be varied from high values of about 0.95 down to 0.6 depending upon the compacting load. A fourth difference between the two systems is the fact that the beds of fibrous media have a much wider pore size distribution. Thus, although the same basic mechanism should apply to the dispersion of solute in both beds of granular or fibrous media, the longitudinal dispersion coefficient will be affected by geometrical differences between the two types of beds.

In order to use the differential equation, used to describe the movement of solute in a bed of packed solids, it is necessary to know the appropriate boundary conditions. Three types of boundary conditions (step, pulse, and sine wave) have been used in studies of granular media. In order to use the step function boundary condition, the input stream should instantaneously change from the solution initially saturating the bed to pure water. Although this ideal is never achieved, a number of investigations have been carried out on beds of granular material under the assumption that the actual change is abrupt enough so that the exit concentration data may be treated with the equations developed for the step change condition. This assumption may be valid for thick beds; however, for thin beds this assumption may be appreciably in error. Thus, rather than assume a step function boundary condition, which would be experimentally difficult to satisfy for fiber beds, the actual quasistep input stream concentration history was used as a boundary condition.

The purpose of this investigation was to study the movement of a solute in a bed of packed solids, during the washing process, with particular emphasis on fibrous systems. Since it was deemed impractical to form thick beds of fibrous media, this study was limited to thin beds of packed solids (glass beads, dacron fibers, and viscose fibers). The glass bead system was primarily used to check out the experimental system and verify the mathematical model. The dacron fiber system was investigated in order to indicate the important variables in a nonporous fiber system and to show the differences between a fiber system and a granular system. The viscose fiber system was investigated in order to indicate the importance of the movement of solute within the fibers. In order to avoid any unnecessary complications, the beds of fibrous media were rendered effectively incompressible by applying an external compacting force on the bed during the washing runs. If the external compacting force is large compared with the fluid drag force, then the fluid drag force will not compact the bed further during the washing run; thus, a uniform porosity should be maintained throughout the thickness of the bed.

EXPERIMENTAL EQUIPMENT

The flow system used in this study is shown in Figure 1. The essential features of this system are a removable sliding valve to form an interface (between the water and the aqueous solution saturating the bed) near the top of the bed, a permeable piston to compact the fiber beds (between the bottom of the sliding valve and the top of the piston), a solenoid valve to start and stop flow, a needle valve to control the flow rate of the wash fluid, and an adjustable outlet level control.

The concentrations of the solution entering and leaving the bed were measured by an optical system consisting of an incandescent light source, a copper sulfate heat filter, a colored glass filter, a collimating lens, beam splitting mirrors, two photocells, an oscilloscope, and a Polaroid Land camera. The

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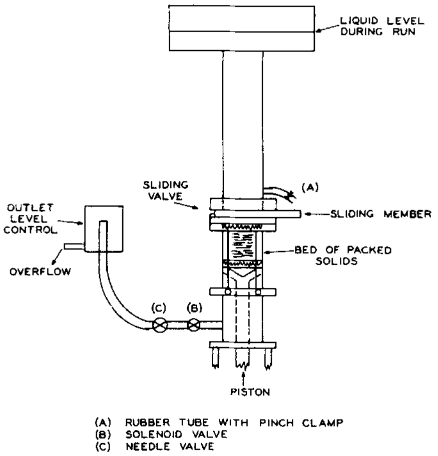

Fig. 1. Schematic diagram of flow system.

bottom of the sliding valve, the piston which compacts the bed, and the tube containing the bed were constructed to allow one beam of light to be passed just above the bed and another beam of light to be passed just below the bed. The details of this construction are shown in Figure 2. By photographing the oscilloscope response to the photocell output, a record of the light intensity was obtained. This was then converted to a concentration vs. time plot by using Beer's law.

BED PREPARATION

In order to form a bed of packed solids, the piston in Figure 1 was lowered and the sliding valve was removed. In the case of the glass bead beds, the bed was formed by pouring the beads into the tube which was nearly full of water. In the case of the fiber beds, the tank was replaced by a pad-forming flow distributor, similar to that described by Ingmanson (5). A dilute slurry of deaerated fibers in deaerated, deionized water, contained in a large stirred tank, was then siphoned into the pad-forming flow distributor. As the water was removed from the slurry, by opening valves (C) and (B), (Figure 1), a fiber bed was formed on the screen above the piston. It was necessary to use a very low consistency in order to avoid flocculation of the fibers in the tube above the piston. Under these conditions a bed consisting of randomly orientated fibers lying horizontally should be obtained. After a fiber bed was formed, a screen was placed above the bed, the sliding valve was replaced, and the fiber bed was compressed against the bottom of the sliding valve by raising the piston.

WASHING RUN

The beds were prepared for a washing run by cycling a diacetyl solution through the bed until the concentration of the diacetyl solution was unchanged by further contact with the bed. The level of the solution was then lowered until it coincided with the center of the sliding valve. The sliding valve was then closed, and the tube above the valve and the tank were filled with water. The needle valve was then adjusted so that the proper flow rate would be obtained during the washing run. After the outlet overflow was connected to a collection vessel and the timer was activated, the apparatus was ready for a washing run.

Just prior to the washing run, the camera, connected to the oscilloscope, was opened. The sliding valve was then opened by pushing the sliding member against a stop. As soon as the sliding valve was opened, a microswitch was released which opened the solenoid valve, started the oscilloscope sweep, and started the timer. At the end of the run, the camera was closed, and the photograph of the oscilloscope response was developed.

The flow rate during the run was calculated from the volume of liquid which passed through the apparatus and the duration of the run. After several runs at different flow rates, the bed was removed from the apparatus, dried at 105°C., and weighed.

THEORY

Although a complete mathematical description of the complex flow pattern in a bed of randomly oriented solid particles is not presently available, the overall movement of a solute, in a bed of nonporous granular material, has been described (*2, 4, 7, 9, 12, 15*) with the diffusionlike differential equation:

$$\frac{\partial c}{\partial t} = -u\frac{\partial c}{\partial z} + D_L\frac{\partial^2 c}{\partial z^2} \tag{1}$$

In the above equation the molecular diffusion coefficient (D_v) is replaced by a longitudinal dispersion coefficient (D_L) which is a function of the flow pattern within the bed (unless very low flow rates are used, D_v is small compared with D_L and may be neglected).

If a bed of porous solids is used, then an additional term must be added to Equation (1) to account for the accumulation (or depletion) of material sorbed by the solid. Equation (1) thus becomes:

$$D_L\frac{\partial^2 c}{\partial z^2} - u\frac{\partial c}{\partial z} = \frac{\partial c}{\partial t} + \frac{1-\epsilon}{\epsilon}\frac{\partial \eta}{\partial t} \tag{2}$$

Before Equation (2) proposed by Lapidus and Amundson (*8*) can be solved, it is necessary to express η in terms of c. This may be done in a number of ways, depending upon the system in question; however, the authors will consider only those cases where

$$\eta = KC \tag{3}$$

If Equation (3) applies, $\partial\eta/\partial t = K(\partial c/\partial t)$, and Equation (2) becomes

$$D_L = \frac{\partial^2 c}{\partial z^2} - u\frac{\partial^2 c}{\partial z} = \frac{1}{\lambda}\frac{\partial c}{\partial t} \tag{4}$$

where

$$\lambda = 1/(1 - K + K/\epsilon)$$

Lapidus and Amundson (*8*) have solved Equation (4) for the case where the boundary condition is a step function. Their solution is

$$\frac{c}{c_0} = 1 - 1/2\left[\operatorname{erfc}\left(\frac{1-R\lambda}{2\sqrt{R\lambda S}}\right) + e^{1/S}\operatorname{erfc}\left(\frac{1+R\lambda}{2\sqrt{R\lambda S}}\right)\right] \tag{5}$$

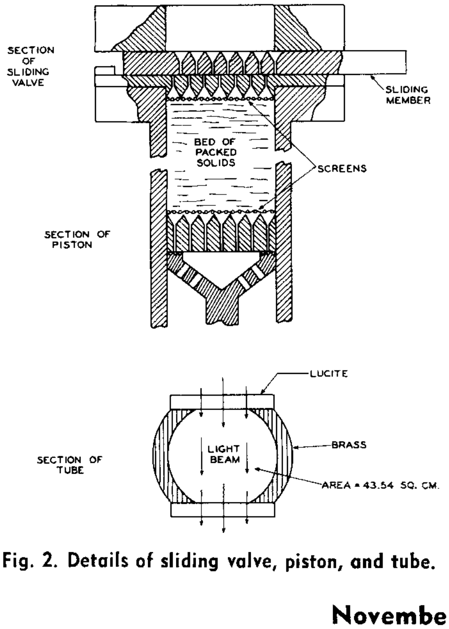

Fig. 2. Details of sliding valve, piston, and tube.

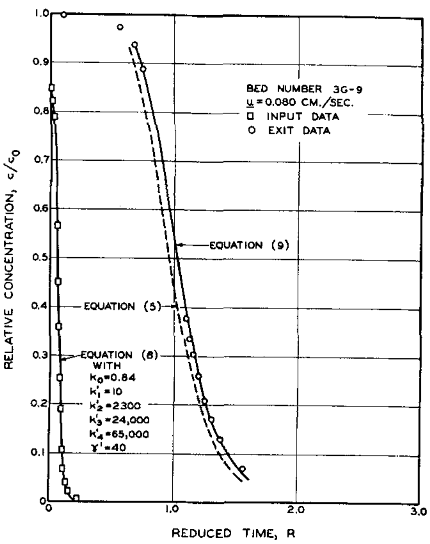

Fig. 3. Washing run data and correlation for glass beads.

In order to avoid the experimental difficulties involved in obtaining a satisfactory step function (which is particularly difficult for thin beds such as must be used with fibrous systems), the following boundary conditions were used in this study:

$$c = c_0, \text{ for } z > 0, \text{ at } t = 0 \tag{6}$$

$$c = f(t) = c_0\,(k_0 + k_1 t + k_2 t^2 + k_3 t^3 + k_4 t^4)\,e^{-\gamma t}$$

$$\text{at } z = 0,\ t > 0 \tag{7}$$

or

$$c = f(R) = c_0\,(k_0 + k_1' R + k_2' R^2 + k_3' R^3 + k_4' R^4)\,e^{-\gamma' R} \tag{8}$$

Equation (7) is an empirical representation of the concentration at $z = 0$, as a function of time, in which the constants k_0, k_1, k_2, k_3, k_4 are adjusted so that Equation (7) agrees with the experimentally determined curve of c vs. t. Equation (8) is Equation (7) rearranged in dimensionless form.

The solution of Equation (4) with the above boundary conditions was obtained by using Laplace transforms (*14*). The solution, in terms of the dimensionless quantities c/c_0, $R = ut/z$, and $S = D_L/uz$, is

$$\frac{c}{c_0} = 1 - 1/2\left[\operatorname{erfc}\left(\frac{1 - R\lambda}{2\sqrt{R\lambda S}}\right) + \right.$$

$$\left. e^{1/S}\operatorname{erfc}\left(\frac{1 + R\lambda}{2\sqrt{R\lambda S}}\right)\right] +$$

$$\frac{1}{\sqrt{\pi\lambda RS}}\int_1^{\infty}[k_0 + k_1'\omega_1 + k_2'\omega_1^2 + k_3'\omega_1^3 + k_4'\omega_1^4]$$

$$\exp\left[\frac{1}{2S} - \gamma^1\omega_1 - \frac{R\lambda}{4S\omega^2} - \frac{\omega^2}{4R\lambda S}\right] d\omega \tag{9}$$

where

$$\omega_1 = \left(1 - \frac{1}{\omega^2}\right)R$$

The advantage of Equation (9) is that it is possible to obtain the same information (generally D_L is the unknown to be determined from the experimental data) as may be obtained from the step-function solution without the experimental difficulties which are encountered in obtaining a step function. This is particularly important for thin beds, such as those used in this study.

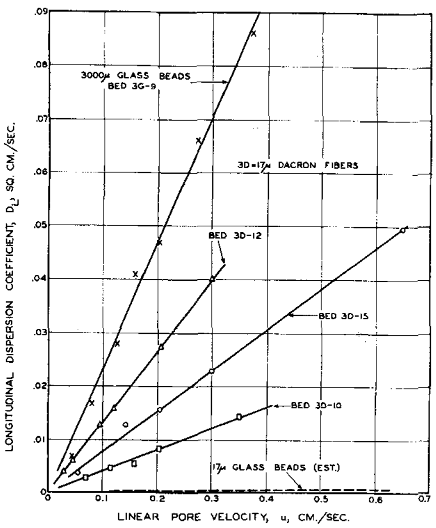

Fig. 4. Dependence of dispersion coefficient on velocity.

TREATMENT OF DATA

The first step in the treatment of the data consisted of converting the photograph of the oscilloscope trace to a plot of c/c_o vs. R, for both the input and exit streams, by using Beer's law. A material balance was then carried out on the system in the following manner. The area under the exit curve (c/c_0 vs. R) minus the area under the input curve, multiplied by the void volume of the bed, equals the volume of solution, at a concentration c_0, which was removed from the bed during the washing run. This, therefore, should represent the void volume of the bed plus the void volume of the particles in the bed.

The next step in the data reduction consisted of adjusting the constants in Equation (8) so that Equation (8) was in reasonable agreement with the plot of c/c_0 vs. R for the input curve. Once the constants in Equation (8) were chosen, the only unknown in Equation (9) was S. In order to determine S, Equation (9) was plotted for several values of S until a satisfactory fit was obtained between Equation (9) and the plot of c/c_0 vs. R for the exit curve.

RESULTS

The results of a typical washing run on the 3-mm. glass bead system are shown in Figure 3.

Figure 4 shows the longitudinal dispersion coefficient D_L as a function of fluid velocity for the 3-mm. glass bead system and for three different nonporous dacron fiber beds. An estimated curve (with the same Peclet number assumed as the 3-mm. glass bead bed) for a 17-μ glass bead bed is also shown in Figure 4.

The properties of the above beds, as well as some viscose fiber beds, are shown in Table 1. The mixing parameter D_L/u and the range of fluid velocities over which it was determined are also included in Table 1. Although the dacron beds varied in porosity from 0.8 to 0.9, the indicated variation in mixing parameter was more likely because of other minor variations in bed structure.

The results of some runs on beds of porous viscose fibers are shown in Figures 5 and 6.

DISCUSSION

A glass bead system was used to check out the experimental system and verify the validity of Equation (9). A dacron fiber system was investigated in order to indicate some of the important variables in a nonporous fiber sys-

Table 1. Bed Characteristics

Bed	Material	Diameter	ϵ	u, cm./sec.	z, cm.	λ	D_L/u, cm.
3G-9	Glass beads	3-mm.	0.45	0.043 to 0.374	5.29	1.0	0.233
3D-10	Dacron fiber	17 μ	0.916	0.068 to 0.352	5.29	1.0	0.040
3D-12	Dacron fiber	17 μ	0.885	0.027 to 0.300	5.29	1.0	0.131
3D-15	Dacron fiber	17 μ	0.815	0.055 to 0.647	5.32	1.0	0.076
1V-11	Viscose fiber	16 μ	0.808	0.080 to 0.329	3.05	0.892	0.0764
1V-14	Viscose fiber	16 μ	0.817	0.520	5.41	0.882	0.0764
64V-13	Viscose fiber	120 μ	0.803	0.042 to 1.316	5.39	0.876	0.077

tem and to show some of the differences between a fiber system and a system of granular material. Viscose fiber systems were investigated in order to indicate the importance of the movement of solute within the fibers.

In the case of the glass beads and the dacron fibers, no sorption of diacetyl occurred; thus η was zero, and λ in Equation (9) was 1. In the case of the viscose fibers, however, sorption of diacetyl did occur; thus η was not equal to zero. If equilibrium exists between the fiber and the surrounding solution (very rapid diffusion within the fiber), and the only solute in the fiber is contained in solution absorbed by the swollen fiber, one may write

$$\eta = \epsilon_f c \tag{10}$$

Thus

$$\lambda = 1/(1 - \epsilon_f + \epsilon_f/\epsilon) \tag{11}$$

The above value of λ was used in Equation (9) for the viscose fiber beds.

From Figure 3 one can see that Equation (9) adequately describes the exit concentration data. One can also see that the step function approximation, as represented by Equation (5), would not be satisfactory. The fact that Equation (9) (with a constant value of D_L/u) adequately describes the exit concentration data for both the thin viscose fiber bed (IV-11) and the thick viscose fiber bed (IV-14), (see Figure 5), indicates that end effects are properly accounted for by Equation (9).

The close agreement between Equation (9), with λ given by Equation (11), and the experimental data in Figure 5 indicates that Equation (10) is correct. That is, the assumption of equilibrium between the fiber and solution was adequate, and absorption occurred whereas adsorption did not.

A number of workers (*1-3, 6, 12, 14*) have predicted that the mixing parameter D_L/u should be constant for any given bed. Experimentally, Raimondi et al. (*12*), Ebach (*4*), and Liles et al. (*10*) found that D_L was proportional to u, $u^{1.08}$, and $u^{0.83}$, respectively. From Figures 4 and 5 one can see that D_L was proportional to u for each bed in the glass bead system, the Dacron fiber systems, and the viscose fiber systems.

The fact that Equation (9) adequately describes the exit concentration data, that D_L/u is constant for each bed, and that the value of D_L/u for the glass bead bed is in general agreement with that of other workers [for example, Raimondi et al. (*12*)] are offered as evidence that this investigation has developed the equations [Equation (9) based on the boundary condition expressed by Equations (6) and (7)] and techniques necessary to describe and measure the washing of their beds of packed solids for which the step function boundary condition is not satisfied.

The precision of determining S ($= D_L/uz$), for different runs on the same bed, was about 10%. Since a change in S of about 10% was necessary in order to appreciably change a given curve, represented by Equation (9), a precision of 10% was considered satisfactory.

The fact that Equation (9) adequately describes the data for both the Dacron fiber beds and the viscose fiber beds (when the assumption of equilibrium is adequate), and the fact that D_L/u is a constant for each fiber bed, indicates that the same basic mechanism of fluid particle dispersion applies to beds of fibrous media as that which applies to beds of granular media; however, physical differences in the two systems cause the mixing parameter D_L/u to behave differently in the two systems.

One cause of fluid particle dispersion is the fact that under the same pressure gradient, fluid particles in large pores will travel more rapidly than fluid particles in small pores. Thus, D_L/u, which is a measure of fluid particle dispersion, will depend upon the pore size distribution in a

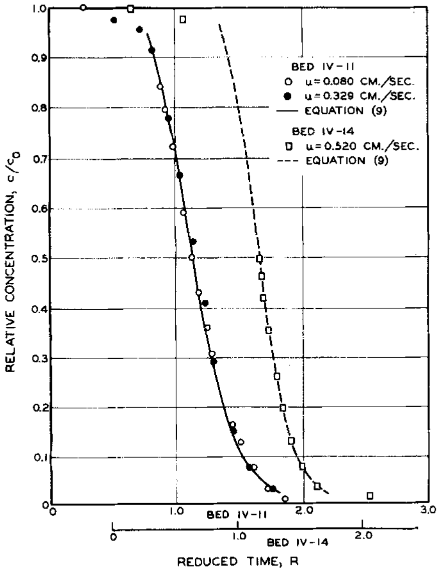

Fig. 5. Washing run data and correlation for viscose fibers.

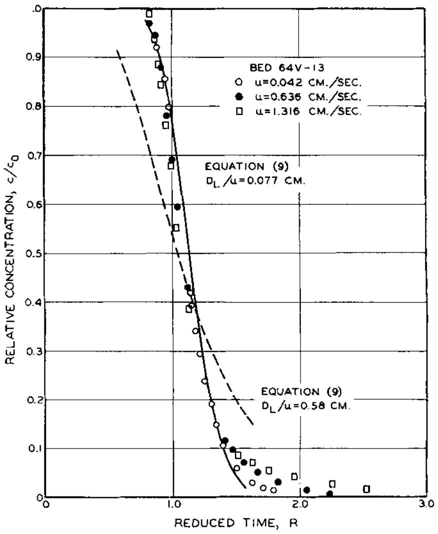

Fig. 6. Washing run data and correlation for viscose fibers.

bed of packed solids. If the pore size distribution is wider, then velocity variations will be greater and D_L/u will be larger. Raimondi et al. (*12*) found that D_L/u was about 200 times larger for a sandstone than in packings of uniform size granular particles having the same permeability (same equivalent particle size). They stated that "This difference is due entirely to the wider pore size distribution of natural sandstones." Orlob and Radhakrishna (11) studied the effect of entrained air on D_L/u and found that the ratio of D_L/u for no entrained air to D_L/u for 15% entrained air was 30. Since the entrained air was preferentially trapped in the small pores, it effectively decreased the pore size distribution and thus decreased D_L/u. It is reasonable to believe that the pore size distribution of a 17-μ fiber system is wider than the pore size distribution of a 17-μ bead system. Likewise, the average pore size in a 17-μ fiber system will be greater than in a 17-μ glass bead system. Both of these facts indicate that D_L/u should be greater for the 17-μ fiber beds than for the 17-μ glass bead system. From Figure 4 one sees that D_L/u is much greater for the three fiber beds than for the hypothetical 17-μ glass bead bed (calculated from 3-mm. glass bead data with the assumption of a constant Peclet number).

In a more detailed theoretical discussion (*14*) it was predicted that when dynamic similarity is maintained, D_L/u should be proportional to the particle diameter. Ebach (*4*) and Raimondi et al. (*12*) found this to be true, but Liles et al. (*10*) did not find a linear relationship (that is, D_L/u was proportional to $d_p^{0.73}$ for three particle sizes). If D_L/u is proportional to the fiber diameter, then D_L/u for the 64-denier (120 μ) viscose should be eight times the value for the 1-denier (15 μ) viscose fibers. On this assumption, Equation (9) has been plotted in Figure 6 with $D_L/u = 0.58$. It is quite apparent from Figure 6 that this assumption is incorrect. Equation (9) was then plotted with D_L/u the same as in the case of the 1-denier fiber (that is, $D_L/u = 0.077$). This curve adequately represents the data.

The fact that D_L/u is apparently independent of the fiber diameter in these beds may, at first, seem surprising, since a strong relationship between D_L/u and particle size has been demonstrated in the case of granular materials (for example, *3, 4, 10, 12, 13*). In theorizing a direct proportionality between D_L/u and d_p, it is generally assumed that geometric similarity is maintained as d_p is changed. In the case of spherical particles (granular materials), the particles are always geometrically similar, and with reasonable care the packing will also be similar. Thus, geometrically similar beds may be obtained as the particle size is changed, and it is found that D_L/u is proportional to d_p. In the case of the two fiber beds (IV-14 and 64V-13), however, not only is it doubtful that the packing of the particles was the same, but the particles were not truly similar (although the diameter ratio was 8:1, the length ratio was about 1:1). Since geometric similarity cannot be assumed for these two fiber beds, it should not be surprising that the direct proportionality between D_L/u and d_p breaks down. It would appear that the packing (which is probably a much stronger function of fiber length than of diameter), wide pore size distribution, and high porosity (both of which are primarily due to the particle shape, that is long cylinders vs. spheres) define the dispersion characteristics of a fiber bed, and the effect of the fiber diameter is small if not negligible.

From Figure 4 one sees that although D_L/u is constant for each bed, the mixing parameter D_L/u is different for the three Dacron fiber beds. Since the porosity of these beds is different, one might expect this to explain the differences in D_L/u; however, since D_L/u first increases and then decreases as the porosity decreases, it seems unlikely that the porosity is the controlling factor. A more logical explanation would appear to be that the packing of these three beds was different and that the effect of porosity is small in comparison with the effect of the changes in packing.

Although diffusion within the fiber could not be measured directly (*14*), the effect of this process can be seen by comparing the experimental washing curves with the theoretical curve with equilibrium assumed between the fiber and the surrounding solution. Since Equation (9) is in fairly good agreement with the experimental data in Figure 6, the assumption of equilibrium between the surrounding solution and the fiber appears to be fairly good at this fluid velocity (0.042 cm./sec.). However, as the pore velocity is increased, a perceptible breakdown of this assumption is apparent. For example, the data for $u = 1.316$ cm./sec. shows a definite tail. Thus, with the large diameter fibers (that is, 120 μ) the assumption of equilibrium is adequate for low flow rates, but at higher flow rates the diffusion of material from the fibers is not fast enough to keep up with the rapid change in concentration of the surrounding solution.

In the case of solutes which diffuse more slowly within the fibers than the solute used in this study, the tail which occurred in Run 13-6 would occur at lower flow rates. Thus, for solutes which diffuse slowly within the fibers, or for high flow rates, Equation (9) will not be adequate. This is due to the fact that the expression, represented by Equation (10), for the quantity of material sorbed by the fiber is not adequate. In this case, the term $\partial\eta/\partial t$ must be obtained from the solution of the differential equation for the diffusion of solute from the fibers under the appropriate boundary conditions.

SUMMARY AND CONCLUSIONS

An investigation of the movement of a soluble material during the washing of a number of beds of packed solids was carried out. The solids studied included nonporous glass beads, nonporous Dacron fibers, and porous viscose fibers.

The commonly used one-dimensional, diffusionlike, differential equation, in which the molecular diffusion coefficient is replaced by a longitudinal dispersion coefficient, was used to describe the movement of solute in these beds. In order to avoid the experimental difficulties encountered in obtaining a satisfactory step function, the boundary conditions were modified to include the actual quasistep function input concentration history in the form of an empirical equation.

The mixing parameter D_L/u was found to be a constant for all runs on any single bed regardless of the particles used to pack the bed. The value of D_L/u (0.233 cm.) for the 3-mm. glass bead bed is in general agreement with that determined in other investigations.

The same basic mechanism of fluid particle dispersion applies to beds of fibrous media as that which applies to beds of granular media; however, the physical differences in the two types of systems cause the mixing parameter D_L/u to behave differently in the two systems.

The differences in geometry, that is in average pore size and in pore size distribution, between a bed of fibrous media and a bed of granular media of the same particle diameter is believed to account for the fact that D_L/u for the Dacron fiber beds was about sixty times as great (ratios of 100, 60, and 30 were obtained for the three beds) as it would be for a bed of granular media of the same diameter.

In beds of granular media of uniform size particles, where geometrical similarity exists, the mixing parameter D_L/u is generally proportional to the particle diameter. In this study the fiber diameter was changed by a factor of

8 without a noticeable change in D_L/u. On the other hand, the three Dacron fiber beds, with a fixed particle diameter, had widely different values of D_L/u. Although the porosity was different for each of these beds, it is believed that the observed difference in D_L/u was probably due to a difference in packing, or formation, of the beds rather than the difference in porosity. Thus, it appears that a single parameter, such as particle diameter, does not define a bed of fibrous media since geometric similarity is not necessarily maintained from bed to bed.

The theoretical equation for the washing of the viscose fiber beds was written with equilibrium assumed between the fibers and the surrounding solution. The assumption of equilibrium was found to be satisfactory over the range of flow rates investigated for the 1-denier fibers (16-μ diameter). For the 64-denier (120 μ) fibers, the assumption of equilibrium was satisfactory at low flow rates; however, at higher flow rates, this assumption was not satisfactory. In this case the experimental curve exhibited an appreciable tail due to the relatively slow intrafiber diffusion of the solute.

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NOTATION

c = concentration of fluid, g. solute/cc. solution
c_0 = concentration initially in bed at $t = 0$, g. solute/cc. solution
D_L = longitudinal dispersion coefficient due to flow pattern, sq. cm./sec.
D_L/u = mixing parameter, cm.
D_v = longitudinal molecular diffusion coefficient, sq. cm./sec.
d_p = particle diameter, cm.
K, K_0 = constants in Equation (3)
k_0, k_1, k_2, k_3, k_4 = constants in Equation (7)
k_1', k_2', k_3', k_4' = constants in Equation (8)
N_{Pe} = Peclet number = ud_p/D_L
R = ut/z
S = D_L/uz
t = time, sec.
u = average linear pore velocity, cm./sec.
z = longitudinal distance variable, cm.

Greek Letters

γ = constant in Equation (7)
γ' = constant in Equation (8)
ϵ = bed porosity
ϵ_f = porosity of fiber
η = quantity of solute sorbed by the solid per unit volume of solid, g. solute/cc. solid
λ = $1/(1 - K + K/\epsilon)$
ω = a variable of integration = $\sqrt{t/\tau}$
ω_1 = $[1 - (1/\omega^2)]R$

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Vapor-Liquid Equilibrium in the System Bromine Pentafluoride–Uranium Hexafluoride

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The system bromine pentafluoride-uranium hexafluoride is of interest in various fluoride volatility methods for the processing of nuclear fuel materials. These volatility methods are commonly based on distillation to purify and recover the uranium as the hexafluoride compound (*1*).

Two equilibrium studies have previously been published on the bromine pentafluoride-uranium hexafluoride system. One study was on the solid-liquid equilibrium; the results showed a simple eutectic at 63.5°C. at a composition of 3.2 mole % uranium hexafluoride (*2*). The solubility of solid-uranium hexafluoride in the liquid-bromine pentafluoride was stated to be essentially ideal (*2*). The other study gave some measurements on the vapor-liquid equilibria in the bromine pentafluoride-uranium hexafluoride system as obtained on a static type of apparatus; the results indicated that binary was nonideal and an azeotrope was reported at 90 mole % bromine pentafluoride (*3*).

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